Kindergarten Students’ Organization of Counting in Joint Counting Tasks and the Emergence of Cooperation

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The purpose of the study was to investigate and document possibilities for and manifestations of collaborative work with pairs of kindergarten students while they worked on tasks designed to promote early number development. Ten students, paired to be compatible with respect to their development of counting, were taught weekly for a period of 4 months. The students were addressed as pairs and provided with only one set of counting materials. The students generated 4 strategies to organize their counting: counting parts side by side, counting all at the same time, taking turns, and counting cooperatively. Cooperative counting was defined as a counting episode in which the counting acts of both students merged into a single activity with the partners working toward a common goal. Three themes emerged from the analysis of the cooperative solutions: (a) the relation between cooperation and the specific requirements of a counting task, (b) the relation between the ability to work cooperatively and the students’ development of counting, and (c) the students’ need to complement a cooperative venture with a solution of their own.

Key Words: Cooperative learning; Early number learning; Children’s strategies; Cognitive development; Constructivism; Early childhood K-4

Cooperative learning in small groups—in comparison with individualistic and competitive learning arrangements—has been studied and described extensively (e.g., Davidson, 1990; Johnson & Johnson, 1991; Slavin, 1983, 1988, 1990, 1991; Wood & Yackel, 1990; Yackel, Cobb, & Wood, 1991). Collaborative work among young children, however, has not received as much attention as cooperation among older students. Neither Slavin (1983, 1990, 1991) nor Webb (1991) mentioned any studies of cooperation with students before second grade. The overview of Johnson and Johnson (1985) and the annotated bibliography of Totten and collaborators (Totten, Sills, Digby, & Russ, 1991) referred to only a few studies with first graders or younger children.

Among those studies is Azmitia’s (1988) investigation of solitary and collaborative performances in a construction task. Preschool children were asked to build replicas of Lego models and were categorized as experts or novices depending on their performances. Azmitia found that for the experts and for those novices who worked alone or with another novice, performances did not improve over two experimental sessions of replicating Lego models. Novices in the mixed-pair condition, however, significantly improved their performances during the interactive sessions, and they maintained the improvement during the posttest session. Azmitia identi-
fied observational learning and guidance by the expert as mediating the novice’s progress. Cognitive conflict, defined as a sequence of at least three statements reflecting disagreement about a step in the task, did not play a significant role. In contrast, Perret-Clermont (1980) saw cognitive conflict, defined as the experience of a different point of view, as the essential element contributing to the fact that nonconserving children made more progress toward conservation when they worked with two conservers than when they worked with intermediate children or with nonconserving children. As in Azmitia’s study, the most advanced children had the least opportunity for progress; they had already attained mastery level with respect to the given task.

In contrast to Azmitia’s (1988) and Perret-Clermont’s (1980) experimental studies, the study reported here was conceptualized as an observational, process-oriented investigation, and the interactive working condition was taken as given. The purpose of the study was to investigate and document possibilities for and manifestations of collaborative work with pairs of kindergarten students while they worked on tasks designed to promote early number development. The students were generally addressed as a pair and were provided with only one set of counting materials. Thus, the joint counting tasks implicitly established the need for some form of collaboration for the students in each pair.

In the literature, the terms collaboration, cooperation, and small-group learning are frequently not defined (e.g., Johnson & Johnson, 1991) and are often used interchangeably (Davidson, 1990). In contrast, I differentiate between collaboration and cooperation. With collaboration I refer to the students’ general ability and willingness to work together. For example, taking turns, including a student’s ability and willingness to wait for his or her turn, is seen as a form of collaboration. My definition of cooperation is more restrictive: For a joint counting activity to be characterized as cooperation, the students’ actions must merge into a single activity with both students working toward a common goal. This definition builds on Deutsch’s (1949) description of a cooperative social situation as one in which the goals for each individual are such that “a goal-region can be entered … by any given individual or sub-unit only if all the individuals or sub-units under consideration can also enter their respective goal-regions” (p. 132). The requirement that the goal be a common goal goes beyond Deutsch’s definition and includes Mead’s (1937) distinction between cooperation and helping:

In cooperation, the goal is shared, and it is the relationship to the goal which holds the cooperating individuals together; in helpfulness, the goal is shared only through the relationship of the helpers to the individual whose goal it actually is. (p. 17)

With respect to the pair working arrangement, the design and conduct of the study were influenced by the models for small-group instruction developed by Kamii (e.g., Kamii, 1982; Kamii & DeClark, 1985) and by Cobb and collaborators (e.g., Wood, Cobb, & Yackel, 1990; Wood & Yackel, 1990; Yackel et al., 1991). Basic to these models are emphases on problem solving, on meaningful strategies and solutions, and on mutual agreement between the partners. To facilitate mutual agreement and meaningful strategies and solutions, the students participating in the study
were paired homogeneously rather than heterogeneously. With respect to the development of counting, the study built on the framework developed by Steffe and collaborators (e.g., Steffe, 1992; Steffe & Cobb, 1988; Steffe, von Glasersfeld, Richards, & Cobb, 1983) and extended by Wright (1989). In particular, the constructs of counting scheme and counting stage were fundamental in the interpretation and categorization of the students’ counting activities.

To set the stage, I summarize the students’ development of counting over the duration of the teaching sessions and define and present the different forms of collaboration that evolved during the students’ work on joint counting tasks. Cooperation, as a particular form of collaboration, is attended to in more detail. Three themes emerged from the analysis of the cooperative situations: (a) the relation between cooperation and the specific requirements of a counting task, (b) the relation between the ability to work cooperatively and a student’s development of counting, and (c) the students’ needs to complement a cooperative venture with solutions of their own.

METHOD

Participants

The study was conducted with five pairs of kindergarten students. The 10 students (5 girls, 5 boys; 3 African American, 1 Hispanic, 6 White) attended a kindergarten class of 17 students in a public elementary school in a university town in the southeastern United States. On the bases of classroom observations and an analysis of initial interviews, participants were selected according to three criteria: (a) a student’s general ability to work with a partner; (b) a student’s willingness to work with me; and (c) the student’s development of counting, because the design of the study required students from different counting stages. The pairs were assembled so that the students in each pair were as compatible as possible with respect to their development of counting. Table 1 gives an overview of the participating students, their ages at the beginning of the teaching sessions, the instructional time for each pair, the reading group attended by each student (with Group A denoting the more advanced group), and the initial and final counting stages for each student. (The counting stages are discussed in a later section.)

Data Collection

The data collection lasted from November 1990 to May 1991 and consisted of interviews with individual students and of teaching sessions with pairs of students, both of which took place in the morning while the other students attended a reading group or worked in an independent learning center. The sessions were conducted in the hallway outside the classroom and were videotaped and audiotaped.

Students were individually interviewed three times: before the teaching sessions began, before the spring holidays, and after the teaching sessions ended. The tasks in the initial interviews addressed each student’s command of the standard number-word sequence, his or her ability to count unscreened and partly screened collections, and the ability to recognize and represent spatial patterns. The interviews
conducted before the spring holidays were designed as individual teaching sessions with specific activities for each student. The final interviews consisted of the items from the initial interview and of additional tasks reflecting each pair’s work during the teaching sessions.

Typically, a pair of students participated in a teaching session once a week for 20 to 45 minutes, with the length of each session dependent on the length and nature of the concurrent classroom activity and on the students’ attention spans. The sessions were divided into several segments, with activities highlighting different aspects of early number learning: activities focusing on counting, activities involving visual and auditory patterns, and activities focusing on the order of the number-word sequence. Usually a game concluded the session. The activities and games were varied according to the counting stages of the students.

**Tasks**

**Counting tasks.** The students were asked to complete counting tasks involving unscreened collections (Figure 1a), partly screened collections (Figures 1b, c), or two screened collections (Figure 1d). The counting tasks were modeled after tasks developed by Steffe and collaborators (Steffe & Cobb, 1988; Steffe et al., 1983; Wright, 1989). The sizes of the screened and unscreened parts were varied according to the students’ counting stages. In missing-item tasks (Figure 1c), the students usually counted the whole collection before part of it was covered.

**Tasks for consolidating the number-word sequence.** Activities for refining and extending the number-word sequence supplemented the counting activities on a regular basis. Two linear models were used, a structured number line (Figure 2a) and number lines consisting of linearly arranged stickers (Figure 2b). The students were
asked to place wooden balls on specific pegs of the structured number line or to attach numeral labels to items of the sticker line. The different heights of the fifth and tenth pegs of the structured number line and the initial numeral stickers placed on the sticker line served as benchmarks that fostered the entry of the number-word sequence at places other than one. The number word activities also addressed the relations after, before, and between.

*Tasks involving patterns.* Because visual and auditory patterns can complement counting activities in the construction of number (Steffe & von Glasersfeld, 1988), pattern cards displaying standard (domino) and nonstandard patterns (e.g.,
Figures 3a–c) were used in some of the counting tasks and games. In addition, students were asked to identify a pattern after a brief exposure or to reproduce a certain configuration after they had looked at it closely.

![Figures 3a–c)](image)

**Figure 3.** Working with patterns.

**Games.** The games ranged from simple counting games, such as Hop To It (Hughes, 1986), to games involving visual patterns and counting, such as War, Double War, Go Fish (Kamii & DeClark, 1985), and Peek Through the Wall (Baratta-Lorton, 1976), to games designed to encourage students to look ahead and to anticipate possible results of an action, for example, Magic Number (Wiegel, 1993).

**The Role of the Teacher**

With respect to the students’ development of counting, the primary role of the teacher was that of posing tasks. Teacher interventions during or after a counting episode depended on the demands of the task, the students’ actions, and their counting stages. For example, for most students it was not yet “obvious and logically necessary” (Piaget, 1941/1961, p. 61) that counting a collection more than once should yield the same result each time. Because working toward agreement between the partners was an important component of the pair working arrangement, the teacher intervened by summarizing the results of both students and occasionally by encouraging them to count again. As the teaching sessions progressed, these interventions became less necessary because the students were more aware of their own counting as well as of their partners’. Interventions were unnecessary for the two students with the most advanced counting schemes. From the beginning, they initiated a new round of counting whenever they arrived at discrepant results.
The metaphor used to introduce the pair working condition was that of teamwork. At the beginning of each pair's first teaching session, the students were asked whether they knew what a team was. Most students connected the term *team* with sports in some way. In the context of the teaching sessions, teamwork was defined as “working together and not quarreling.” After the first session, the metaphor of teamwork was used casually by teacher and students to refer to the obligation to work together.

**Analysis**

After each teaching session, I recorded the students’ actions and interactions in a written protocol, on the basis of at least two viewings of the videotape. After the completion of the teaching sessions in May 1991, I again viewed all videotapes, in chronological order for each pair of students, and I updated and extended the protocols. The processes of viewing tape segments and of refining the protocols continued throughout the data analysis.

I analyzed the students’ actions as seen on the videotapes and as documented in the protocols according to the students’ development of counting and their social interactions, including their organizations of counting. On the basis of the work of Steffe and his associates (Steffe, 1992; Steffe & Cobb, 1988; Steffe et al., 1983), I developed the following behavioral indicators for counting stages.

*Perceptual counting stage:* The student counts unscreened collections.

*Transition from perceptual to figurative counting stage:* The student counts partly screened collections in some contexts. The student experiences conflict when he or she is unable to keep track of counting acts.

*Figurative counting stage:* The student can count partly screened collections in all contexts. When counting the screened part, the student often keeps track of the counting acts with finger patterns. The student can focus on the screened items to find the number covered.

*Transition from figurative counting stage to initial number sequence:* The student counts on in some contexts and starts to monitor counting during the activity itself.

*Initial number sequence:* The student can count on in all contexts.

*Tacitly nested number sequence:* The student is able to establish the counting acts as countable items; that is, the student can double count to keep track of counting acts.

*Explicitly nested number sequence:* The student can use part-whole reasoning when counting a collection consisting of two or more screened subcollections.

I used analytic induction (LeCompte, Preissle, & Tesch, 1993) to analyze the data on the students’ interactions and their organizations of counting. Analytic induction involves “scanning the data for categories of phenomena and for relationships among such categories, developing typologies and hypotheses upon an examination of initial cases, then modifying and refining them on the basis of subsequent cases” (p. 254).
RESULTS

Development of Counting and Organization of Counting in Joint Counting Tasks

At the beginning of the teaching sessions, four students were in the perceptual counting stage, four students were in transition from the perceptual to the figurative stage, one student was in the figurative stage, and one student had constructed the initial number sequence. At the end of the sessions, all students but one had progressed beyond their initial counting stages. The students’ progress is summarized in the last two columns of Table 1.

A joint counting task was defined as a task in which the students were addressed as a pair and provided with only one set of counting materials. To complete a joint counting task, the students had to come to some kind of agreement—implicit or explicit—on how to organize their counting. The four strategies the pairs of students generated are described in order of their increasing requirement for a pair’s coordination of actions.

Strategy 1: Counting Parts Side by Side

A counting activity with no coordination of actions was classified as counting parts side by side. When counting parts side by side, the students worked by themselves, without much regard to the partner. When asked, for example, to count a collection of blocks “together, as a team,” each student moved blocks from the initial location to his or her place, uttering “One, two, …” until the original collection was exhausted. The outcome of this strategy was two or more subcollections, each of which had been counted by one student but not by the other. Counting parts side by side occurred only in the context of counting unscreened collections and mainly at the beginning of the teaching sessions.

Strategy 2: Counting All at the Same Time

Counting all at the same time differed from counting parts side by side in that the students did not create subcollections, but each student counted all (or most of) the objects in the original collection. The students were still engaged in two separate, mostly independent, counting activities, but they kept the original collection intact while pushing the objects to a common location, moving them only slightly within the vicinity of the original locations, or not displacing them at all. Occasionally, the students in a pair adjusted to the same rhythm both in saying the number words and in moving the blocks.

Whereas counting parts side by side occurred only during work with unscreened collections, the more advanced students used counting all at the same time also in tasks involving partly screened collections. Counting all at the same time emerged if one or both of the partners were beyond the figurative stage and did not need to manipulate the objects to count them. Andrea, for example, having constructed the initial number sequence, could count on and thus frequently did not need to handle the objects; uttering a number word took the place of counting the unscreened part of the collection.
She then kept track of her subsequent counting acts by extending fingers until she had completed the finger pattern signifying the number of screened items.

**Strategy 3: Taking Turns**

_Taking turns_, with only one student at a time handling the objects while the partner waited, was overall the most frequently used organization of counting. The prevalence of taking turns is not surprising for several reasons. First, taking turns was an accepted and encouraged way of social behavior in the students’ school lives. Second, the highly structured activities for consolidating the number-word sequence lent themselves well to taking turns. If a student had placed a ball on a peg of the structured number line (see Figure 2a), it evolved almost naturally that the partner leaned forward and took charge of the next ball. Finally, taking turns was an integral element of most games and as such was part of all but a few teaching sessions.

The interesting aspect of taking turns was not its frequency but the activities of the student waiting for his or her turn. These activities gave an indication of a student’s ability to make sense of the partner’s actions. The activities of the waiting partner included both nonparticipation and varied forms of participation. With respect to his or her increasing level of involvement in and understanding of the partner’s counting, these forms included watching and supporting; watching and joining; solving the partner’s task (at the same time); helping, correcting, and commenting; and completing the partner’s task. During the course of the teaching sessions, students moved from forms of participation with little involvement to those requiring more involvement, with differences across pairs and tasks. I next elaborate on forms of participation that highlight the students’ increasing abilities to make sense of the partners’ actions.

**Watching and joining.** Watching and joining referred to a counting situation in which the student waiting for a turn joined the partner’s counting activity by uttering the number words with the partner. The joining student performed only part of each counting act and did not engage in the rhythmic motor activities that usually aid in coordinating the creation of countable unit items and the production of number words (Steffe et al., 1983). Watching the partner point to or move the items while uttering number words was an opportunity for the student to become aware of his or her own motor activities and to isolate them from the otherwise integrated counting acts. In addition, watching and joining provided a student with the opportunity to experience the partner’s counting as part of his or her own counting activity and thus to make sense of the partner’s actions. Watching and joining can be seen as a first step toward the coordination of two counting activities into a single activity and, thus, as a step toward cooperation.

**Helping, correcting, commenting.** Incidents involving helping, correcting, or commenting occurred most frequently during tasks with unscreened collections. In a sticker-labeling activity involving 20 stickers, for example, Scott pointed to the 19th sticker, saying “Right here,” when Alisha attempted to count from 1. In another case, Adam re-counted 20 stickers: “One,…, eighteen, twenty, twenty-one; see that would be wrong; eighteen, nineteen, twenty.” He made sense of Ben’s diverging result and corrected him at the same time. Comments about the partner’s counting
were sometimes directed to the teacher, such as Scott’s remark, “Her already
counted this,” meaning that his partner had counted an item twice. Counting tasks
involving unscreened collections provided students, especially those in the perceptual
counting stage, with opportunities to interpret the partner’s activities with respect
to their own experiences. These interpretations seemed to be mutually compatible.

Completing the partner’s turn. When a student completed the partner’s count-
ing activity, the counting acts of both students merged into a single counting
episode. I illustrate this merging with an example from Scott and Alisha from Session
9. During a game, Alisha had won 12 beans. The teacher gave her 6 of the beans
and asked, “How many more?” While Alisha counted the 6 beans, Scott watched
and subvocally uttered the last number words with her. When Alisha stopped, he
sequentially extended six fingers while subvocally uttering the number words 7 through
12. He then counted the extended fingers by touching his lips and answered, “Six
more.” An important feature of this episode was that Scott did not have to recount
the six beans on the table but was able to take Alisha’s counting acts (“one,
two,…, six”) as given, substituting them for his own. Scott’s completion of
Alisha’s counting episode was his most advanced form of participation. Again, con-
tinuing and completing another student’s counting episode can be seen as an
important step toward cooperation. It was, however, not yet cooperation because
the two students had not established a common goal.

Strategy 4: Cooperation

A joint counting activity was classified as cooperation if the counting acts of both
students merged into a single counting activity with the students working toward
a common goal. I inferred a common goal if there was some communication in which
“working together” was made explicit. For example, in Session 9, Ben initiated a
cooperative counting episode as follows:

Ben:    Let’s count together, ’cause it might be a little—
       I count first, all right and, and if there’s not enough, you count with your hands,
       all right?
Adam:  Okay—
Ben:    ’Cause we gonna do teamwork.
       (Sequentially extends 5 fingers of his left hand) One, two, three, four, five …
Adam:  (Joins subvocally) Four, five—
Ben:    (Opens his right hand in one movement) There’s ten, all right.
Adam:  I know—
Ben:    Ten … (looks at Adam).
Adam:  (Looks into the air, then at his right hand, sequentially extends his index, middle,
       and ring fingers) Eleven, twelve, thirteen.
Ben:    (With Adam) Eleven, twelve, thirteen.
Adam:  (Moves his right hand— with 3 fingers extended—to a position beside Ben’s 10
       fingers) Okay—

Adam needed some time to tune into his partner’s counting activity. When Ben
started to count, Adam looked down, his head in his hands and a frown on his face, but then subvocally joined Ben’s counting acts by uttering “four, five.” He seemed to need Ben’s prompt of “ten” before he continued the counting activity. Adam’s movement of his three extended fingers to the side of Ben’s hands established his involvement and thus the beginning of the cooperative counting activity.

Cooperative counting occurred during work with unscreened, with partly screened, and with fully screened collections. The incidents of cooperation and the contexts in which they occurred are summarized in Table 2. The five cooperative substrategies are discussed next.

### Cooperative Substrategy 1: Counting alternately
The merging of the students’ individual counting acts into one activity is particularly well illustrated by the strategy of counting alternately. When counting alternately, the students took turns identifying and moving the countable objects and saying the corresponding number words. Counting alternately meant that the two students divided the counting activity between them, with roughly one half of the counting acts performed by one student and one half by the other student.

### Cooperative Substrategy 2: Delegating part of the activity
Delegating part of the activity was a different form of dividing a counting activity between two students. Adam initiated this form of cooperation when Ben wanted to participate in Adam’s turn:

**Ben:** How about I count with you?
**Adam:** You can count what I put in.

In the subsequent counting activity, Adam dropped beans into a box while Ben said the corresponding number words. Delegating part of the activity was different from

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**Table 2**

*Incidents of Cooperation*

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Context</th>
<th>Students (# of incidents)</th>
<th>Session (# of incidents)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Counting alternately</td>
<td>Making a line of checkers&lt;sup&gt;a&lt;/sup&gt;</td>
<td>Andrea/Emily (1)</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>Counting beans into boxes&lt;sup&gt;a&lt;/sup&gt;</td>
<td>Andrea/Emily (1)</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ben/Adam (1)</td>
<td>9</td>
</tr>
<tr>
<td>Delegating part of the activity</td>
<td>Counting beans into boxes&lt;sup&gt;a&lt;/sup&gt;</td>
<td>Ben/Adam (1)</td>
<td>9</td>
</tr>
<tr>
<td>Start and continue</td>
<td>Counting beans into boxes&lt;sup&gt;a&lt;/sup&gt;</td>
<td>Andrea/Emily (1)</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ben/Adam (1)</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>Game: Peek Through the Wall&lt;sup&gt;b&lt;/sup&gt;</td>
<td>Ben/Adam (1)</td>
<td>4</td>
</tr>
<tr>
<td>Sharing fingers</td>
<td>Missing-item tasks</td>
<td>Andrea/Emily (3)</td>
<td>6, 7</td>
</tr>
<tr>
<td></td>
<td>(more than 10 items covered)&lt;sup&gt;b&lt;/sup&gt;</td>
<td>Ben/Adam (3)</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>Counting covered collections (bean boxes, more than 10 beans per box)&lt;sup&gt;b&lt;/sup&gt;</td>
<td>Andrea/Emily (5)</td>
<td>9, 10</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ben/Adam (7)</td>
<td>9, 10, 11</td>
</tr>
<tr>
<td>Double counting</td>
<td>Counting covered collections (bean boxes, more than 10 beans per box)&lt;sup&gt;b&lt;/sup&gt;</td>
<td>Andrea/Emily (2)</td>
<td>10, 11</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ben/Adam (1)</td>
<td>11</td>
</tr>
</tbody>
</table>

<sup>a</sup>Task involves an unscreened collection. <sup>b</sup>Task involves a partly screened collection or two screened collections.

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*Organization of Counting and Emergence of Cooperation*
participating in a partner’s turn by watching and joining. Watching and joining required no explicit communication between the partners; while one partner counted, the other simply joined in by uttering number words. Ben’s question and Adam’s initiation of delegating part of the activity, in contrast, contained that moment of planning and anticipation that established a common goal for the partners.

**Cooperative Substrategy 3: Start and continue.** In a third form of dividing a counting activity, start and continue, one student started by counting part of the objects, and the partner continued by counting the remaining objects. Both students had to be able to relate their own counting acts to those of the partner. The student counting first had to consider the partner’s counting as an extension of his or her own activity. The student continuing the counting episode had to be able to take the partner’s counting as given and substitute it for his or her own counting activity. What distinguished start and continue as a cooperative counting episode from a mere participation in the partner’s turn (e.g., completing the partner’s task) was again the act of planning and anticipation that preceded the actual activity and established the common goal for the students.

**Cooperative Substrategy 4: Sharing fingers.** The most frequently used cooperative strategy was that of sharing fingers. The pair Andrea and Emily and the pair Ben and Adam invented this strategy in the context of a missing-item task with more than 10 objects covered (see next section). The successful completion of this task led to a culture of cooperation in which sharing fingers became routine. Most incidents of sharing fingers occurred in tasks in which the students were asked to count the beans in two closed boxes (Figure 1d), each of which contained 10 or more beans.

**Cooperative Substrategy 5: Double counting.** Double counting was a form of cooperation in which one of the partners counted the covered objects while the other kept track of the number of counting acts that had been performed. The following protocol illustrates cooperative double counting and how one student’s initiation of cooperation was the result of his own futile attempt to keep track of his counting acts.

In Session 11, Ben and Adam were trying to count the beans in two closed boxes, one containing 12 beans, the other 14 beans. To assist in the counting process, the students used a 100-board:

**Ben:** (With his finger on the 12th field of the 100-board) Twelve—(moves his finger to the 13th field), thirteen, (pause) that’s one, thirteen—
Fourteen, that’s two, fifteen-three, sixteen—
(Looks up, then at Adam) You count by one, two, three (slides his finger along the 13th, 14th field—), while I count thirteen, fourteen, all those of it (slides his finger along the fields).
And then when we get up to fourteen, we’ll see what number I get—that I’m pointed to, all right, and I’m right there.

When Ben started counting “thirteen, fourteen…,” sliding his fingers along the respective fields, Adam silently raised his fingers, one after the other. Adam’s nonverbal actions allowed Ben to focus on his own counting acts and on the production of the correct number words without being distracted by a different sequence of words.
Again, the protocol illustrates the act of planning as one of the constitutive elements of the cooperative counting episode. Before the two boys engaged in their cooperative venture, Ben unfolded the complete activity before them. He anticipated and described to Adam two parallel but related counting activities in which the counting acts in one activity referred to the counting acts in the second activity. In this action, Ben established his counting acts as countable items for his partner and thus performed operations characteristic for the tacitly nested number sequence, a counting stage yet to be attained by Ben.

Aspects of Cooperation

As Table 2 shows, the majority of cooperative solutions of joint counting tasks occurred toward the end of the teaching sessions in situations in which more than 10 items had been screened from view. Also, only two of the five pairs, the pair Andrea and Emily and the pair Ben and Adam, solved any of the counting tasks in cooperation. These four students had the most advanced counting schemes; they either had constructed the initial number sequence or were in the process of constructing it. These two observations suggest a relation (a) between task characteristics and the initiation of cooperation and (b) between counting stage and the ability to engage in a cooperative venture. A third theme that evolved from the analysis of the videotapes was the need for individual solutions within a cooperative counting episode.

Relation of Task Characteristics and Cooperation

The size of the covered collection influenced the difficulty of a joint counting task. When counting collections with more than 10 items screened, Ben and Adam, as well as Andrea and Emily, could not keep track of their counting acts because they had not yet constructed finger patterns above 10. The awareness of the inability to keep track led to a perturbation that, in turn, resulted in the initiation of cooperation. For both pairs, the situation leading to a sequence of cooperative counting episodes was a missing-item task with 14 of 17 checkers covered (see Figure 1c). The move toward cooperation occurred after the initial counting activities had failed to produce a satisfactory solution. I briefly describe the initial counting activities and then elaborate on the move toward cooperation.

Ben and Adam initially took turns, with Adam being the first to count. He counted the three visible checkers, then extended in turn all 10 fingers, counting “four, five, …, thirteen,” and finally extended 4 fingers a second time concurrently with counting “fourteen, fifteen, sixteen, seventeen.” He then made an estimate of 12 checkers under the cloth. Adam made this global judgment instead of looking back at the just completed activity and counting the fingers he had extended.

Ben also started his turn by counting the 3 visible checkers and by extending in turn his 10 fingers, counting “four, five, …, thirteen.” But instead of extending some fingers a second time, he folded down 4 fingers, uttering “fourteen, fifteen, sixteen, seventeen.” In contrast to Adam, Ben set out to review how many fingers he had used. He slightly bent the fingers still extended and counted them as “one, two, …,
six.” He then extended the four previously folded fingers, “seven, eight, nine, ten,” stopped, and said in astonishment, “Ten!” He had a disconcerted look on his face. He then made an estimate of 13 checkers under the cloth. Like Adam, Ben had an intuitive awareness that more than 10 checkers were covered. At this point, the teacher suggested they try again, but Ben initiated a cooperative venture instead:

Ben: (Excitedly, to Adam) Hey, how about you count those three—(points to the visible checkers) and I use my hand if you do—want to do the other ones, all right? Then you can count them, and we’ll know.

Ben’s initiative led to a cooperative counting episode in which Adam first counted the three visible checkers, then extended, one by one, his fingers to account for the first 10 checkers under the cloth. Ben continued by extending four of his fingers, then placed his hand, with the four fingers extended, beside Adam’s two hands, and finally counted the fingers they had extended.

Ben’s initiation of pooling fingers seemed to be the result of a conflict between his original result of “ten” and his intuitive awareness that more than 10 checkers were screened. Because of this conflict, Ben’s vague intuition of “more than ten” seemed to become explicit. However, “more than ten” was still not explicit enough because it did not tell how many checkers were covered. Ben knew that counting was a way to make “more than ten” more explicit, as indicated by his comment, “Then you can count them, and we’ll know.” To Ben, initiating the cooperative counting activity seemed to be more accessible than attempting to recount and to monitor his counting acts during the counting activity. Monitoring his counting acts during the activity rather than retrospectively would have required a series of “progressive uniting operations” (Steffe, 1994, p. 158) that were beyond the operations available to Ben at the time.

Emily’s and Andrea’s initiation of cooperation was different from that of Ben in that it evolved out of an implicit request for help. Immediately after the teacher had given the task (Figure 1c), Andrea started to count. Subvocally uttering number words, she extended all fingers of her right hand, then the fingers of her left hand, and then four fingers of her right hand again. She then exclaimed, “There’s not enough for hands!” and reached for one of Emily’s hands.

Emily: (Pulls back, holds up both hands) You can go over on your hands again.

Andrea: I need one of your hands to help me.

Emily: No. You can count over on your hands again.

Andrea: I know, but that’s not what—that’s what’s not helping me.

Like Ben and Adam, Andrea was aware that she had extended more than 10 fingers, and, like Adam, she did not look back and count the fingers she had extended. She experienced a perturbation, indicated by her remark, “There’s not enough for hands!” In contrast to Ben, Andrea did not resolve the situation by making an estimate. Because she was able to identify and verbalize the source of her problem, she set out to solve the problem by reaching for an additional hand. Andrea’s more acute awareness of her actions was also apparent in her reaction to Emily’s initial refusal to give assistance, “I know, but that’s not what—that’s what’s not helping me.”

It is no coincidence that the sequence of cooperative counting episodes was initi-
ated as a result of a perturbation during a missing-item task: Missing-item tasks are more complex than tasks in which students are asked to count all the objects because they require an additional step in which the student must look back at and evaluate the just completed counting activity. Instead of being able to take the last number word uttered as the result of the counting activity, the student has to focus on the second part of that activity, the counting acts referring to the covered items, and has to constitute those counting acts as countable items. Ben’s feeling of uneasiness and resulting need for some modification of his counting scheme, as constituted in the initiation of cooperation, can be seen as the result of the two task characteristics, the size of the covered part of the collection and the complexity of the missing-item format.

Relation of Cooperation and Counting Stage

All cooperative solutions of joint counting tasks were initiated and carried out by students who either had constructed or were in the process of constructing the initial number sequence. None of the students with less sophisticated counting schemes attempted cooperative solutions to any of the counting tasks. This finding does not mean, however, that the students with only perceptual or figurative counting schemes were unable to develop a constructive working relationship. On the contrary, Scott and Alisha, two students with perceptual counting schemes for most of the teaching sessions, had the most peaceful and considerate working relationship of all five pairs. They never argued or were upset with each other. They settled on their own who would start a game or an activity, and they helped each other whenever the situation required. Each frequently participated in the partner’s counting activity by watching and joining. But they did not engage in any cooperative counting episode.

To highlight the apparent relationship between counting stage and organization of counting, I compare solutions of Scott and Alisha with those of Ben and Adam and of Emily and Andrea for the same set of tasks, which were not difficult for any of the students individually because no screened collections were involved. The tasks—counting beans into small cardboard boxes—became challenging only because the pairs were specifically asked to work together “as a team.”

Scott and Alisha: Mutual support. Scott and Alisha organized the bean-counting tasks in the same way that they usually organized their counting: They took turns. Scott took charge of the first box while Alisha waited. After a moment, Alisha reached into the container holding the beans, took out a handful, and presented them to Scott. From this point, Scott counted the beans out of Alisha’s hand.

Alisha’s indirect participation in Scott’s counting activity served to fulfill the teacher’s charge of “working together.” Assuming this supporting role enabled her to be involved in Scott’s activity without directly taking part. Both Scott and Alisha chose this supporting form of participation for the remaining tasks. In all cases, it was the student not engaged in the actual counting activity who initiated the collaboration. Although Scott went beyond supporting Alisha’s activity when he uttered the number words with her and even anticipated the number of beans she needed, he did not engage in a cooperative counting episode with Alisha. The counting acts of the two students...
never merged into one activity. In addition, there was no communication between
the students that might have indicated the establishment of a common goal.

Ben and Adam: Cooperation within taking turns. Ben and Adam, like Scott and
Alisha, organized the bean-counting tasks by taking turns. But within this overall
organization of taking turns, they generated forms of working together that were
more sophisticated than Scott’s and Alisha’s participation through mutual support.
Ben, during his turn, explicitly invited Adam to participate: “You can help me …. Let’s count.” This invitation led to a counting activity that, after some adjustment, proceeded alternately. In the next task, Adam took care of Ben’s desire to partic-
ipate by suggesting that Ben say the number words while Adam dropped the
beans into the box. And in the final task, Ben started the counting activity and Adam
completed it when Ben ran out of beans. Ben’s and Adam’s means of mutual par-
ticipation are examples of cooperation in the sense that their counting acts merged
into single counting activities and were preceded by acts of planning.

Emily and Andrea: Cooperation and taking turns. Emily and Andrea started the
cooperaive venture without much effort. The teacher’s instruction, “Move together
so you can work together,” was sufficient for the girls to engage in a series of count-
ing activities in mutual agreement. They counted alternately in the first task, took
turns for the second and third tasks, and then decided to “do another one together.”
At this point, Andrea noticed that they were working in a pattern:

Andrea: I do one, you do one, we both do one.
Emily: (Joins in) We both do one, you do one, I do one.

Emily’s and Andrea’s exchange reveals a more sophisticated point of view than that
of Ben and Adam. Whereas the boys stayed in the situation of each counting episode,
Emily and Andrea were able to step back from their immediate experience and look
at their actions in context and, as such, from a different perspective. They were able
to view each counting episode in relation to the other episodes and to evaluate their
activity as a whole.

Individual Solutions Within Cooperative Counting Episodes

Most cooperative counting episodes involving covered collections were not
straightforward but consisted of several related counting activities. Usually one of
the students took the lead for part of the cooperative episode, then the other joined
in or took over. Sometimes, a cooperative venture contained several of these
alternating subepisodes, with each student constructing individual solutions of the
task. One example of individual constructions within a cooperative venture was Ben
and Adam’s work on two closed boxes containing 10 and 13 beans.

Adam’s construction of a solution. After Adam had joined Ben’s initial count-
ing activity (see previous section on Cooperation), he intensified his involvement
by twice counting the 13 extended fingers. Both times he counted his own 3
extended fingers first, then the fingers on Ben’s two hands. He closed this part of
the episode with “Okay, we got thirteen all set up,” then reached for the other box:

*Adam:* Thirteen .…

*Ben:* And ten more.

*Adam:* Okay, thirteen, what comes after thirteen, fourteen? Fourteen … (extends his right index finger).

At this point Ben tried to take over; he grabbed Adam’s hand so he would put up his three fingers again. But Adam withdrew and continued:

*Adam:* Fourteen, fifteen, sixteen .…, twenty-three (sequentially extends his 10 fingers). Twenty-three altogether (briefly presents the 10 fingers, then takes the two boxes into his hands). All these, [put] together.

The way Adam emphasized thirteen in his continuation of counting and the way he kept track of his counting acts past 13 strongly suggest that he applied the uniting operation to the 13 previously counted fingers. In this context, the number word thirteen referred to an experiential composite unit whose content was the counted fingers (Steffe & Cobb, 1988). Adam’s determination to resist Ben’s interference and to continue counting emphasizes that his activity was purposeful and that he knew what he was going to do before he did it, namely, to perform 10 counting acts beyond 13.

**Ben’s construction of a solution.** When Adam took charge of the counting activity, Ben stayed involved by trying to reestablish his experiential meaning for 13, the finger patterns of 10 and 3 fingers. But when Adam continued with his actions, Ben turned toward him, watched, and then subvocally joined his partner in saying the number words. As soon as Adam had concluded that there would be “twenty-three all together,” Ben started a new counting activity. He first counted his own fingers as “one, two, …, ten,” then Adam’s as “eleven, twelve, …, twenty,” and finally two of the teacher’s fingers as “twenty-one, twenty-two.”

Having completed this counting activity, Ben immediately started again. He seemed confused when he touched one of his hands, then one of the teacher’s, and then turned to Adam and took his hand. When the teacher noted, “I don’t understand what you are doing, Ben,” he took his head into his hands:

*Ben:* Ah—I need to get ten in my head, then count my tens here (puts his hands on the table, palms up, all fingers extended), then you count thirteen; you guys, you two make thirteen.

Here the teacher’s comment seemed to be helpful for Ben’s organization of his activity. The necessity to give an answer, that is, to verbalize his intentions, helped him clarify for himself what he was trying to do. Asked how to “make thirteen,” Ben explained:

*Ben:* Well make thir—put up three here, put three (takes three of Adam’s fingers and extends them), and this is thirteen (puts his two open hands beside Adam’s three extended fingers).

(To the teacher) You have to—put your two hands out. (To Adam) And you put up three, and now it’s … (counts the extended fingers, including his).

Ben did not have to count to show how to “make thirteen.” Rather, he regenerated the result of the prior cooperative counting experience by first asking for 3 of Adam’s
fingers and then placing his two hands besides those 3 extended fingers. In this regeneration Ben seemed to focus on his 10 and Adam’s 3 fingers separately, one after the other. This experiential juxtaposition of the two finger patterns seemed to constitute his meaning for thirteen.

**Consolidation.** So far, each boy had taken the lead in counting the beans in the boxes at least once, and both had arrived at the same result, 23 beans. Nevertheless, Adam, as soon as Ben had finished, took the initiative and started again. This new initiative eventually led to a counting activity in which both partners participated equally. Adam started with “One, two, …, ten,” extending his fingers, then Ben extended three of the teacher’s fingers, saying, “eleven, twelve, thirteen,” and finally both counted in synchrony, “fourteen, fifteen …, twenty-three,” while Ben kept track with his fingers. After this activity, executed together, both students seemed to experience a sense of closure and were ready to leave this particular task.

**Reflection.** Why did the students have to count repeatedly before they were ready to leave the task? There is no indication that the students counted in response to an external cue, such as an intervention by the teacher. Except for the selection of the boxes and the initial statement of the task, the teacher’s involvement in the episode was minimal. Her role was that of a participant rather than that of a leader. It was the students who took charge of the situation. Their actions, such as the participation in the partner’s counting, the initiation of new counting activities, and the conclusion of the episode at the point of closure, were self-directed and seemed to be independent of the presence of a teacher. The students were active as long as they experienced a perturbation.

Several reasons are possible for the boys’ perturbations and thus for their needs for repeated counting activities within one task. First, the number word *twenty-three* carried no meaning for the students other than as a result of counting. Although the boys were in the process of constructing the initial number sequence, they were not yet able to take a counting activity as given and to substitute a number word for that activity. Note that Adam had to recount the 13 fingers twice to construct thirteen as an experiential composite unit. Similarly, they both had to count all the beans to experience any result as meaningful. Obviously, Ben’s involvement in Adam’s counting activity in the form of watching and joining did not provide a strong enough experience to make *twenty-three* meaningful for him. In addition, Ben could not make sense of Adam’s counting because Ben had not made the same construction as Adam: He had not constructed thirteen as an experiential composite unit. For this reason, Ben had to establish finger patterns for 13 (10 and 3) as well as for ten, and he had to count the contents of both patterns.

Adam, in contrast, did not make enough sense of Ben’s actions to be able to take the result of his counting as given. Instead, Ben’s counting seemed to lead to a new perturbation. To neutralize this perturbation Adam had to count again, and he had to count in a way similar to his first solution. Indeed, he made several attempts to get Ben to establish 13 as a first step. He was satisfied only when there was a clear distinction between making 13 by using Adam’s 10 fingers and 3 of the teacher’s and making 10 by using Ben’s fingers. The last counting activity seemed to satisfy
both students. Ben had all beans represented at the same time, and Adam could make the separation between the 13 beans in one box and the 10 in the other. This last counting activity made sense to both students and thus provided the sense of closure.

It should be noted that the cooperative solutions involving unscreened collections were usually completed in a single counting activity. When they counted visible objects, each student’s contribution to the counting activity could be interpreted by the partner in terms of his or her available scheme and did not lead to a perturbation. Thus, there was no need for repeated counting activities within the cooperative episode.

Beyond Cooperation—The Final Interviews

For the four cooperating students, their cooperative work during the last few teaching sessions was complemented by individual progress apparent during the final interview, approximately a week after the last teaching session. All four students were now individually able to solve missing-item tasks with more than 10 items covered. For most problems, they counted the visible items first, then the covered items by extending a finger for each item counted. In a second step they reviewed the just completed activity by counting the fingers extended. Adam, Ben, and Emily also used a guess-and-check strategy in which they first estimated the number of items covered and then counted on to account for the unscreened items. The students knew that their initial estimate was not correct if they did not reach the number word attached to the whole collection. Finally, Andrea used a third strategy in which she counted backward. For example, with 2 of 23 items visible, she counted “twenty-three, twenty-two, twenty-one,” and concluded that 21 items were under the cover.

In addition, the four students were able to count the beans in two closed boxes with more than 10 beans in each box without the help of the partner. Although Ben and Adam still needed to use the teacher’s fingers for some of the problems, their solutions were more advanced than in prior teaching sessions because they were able to monitor their counting acts during the activity rather than retrospectively. Emily counted on in all cases and did not need any fingers but her own.

Andrea made the most dramatic progress. She abandoned counting on and used part-whole reasoning. Working with two boxes containing 17 and 15 beans, she explained as follows:

Andrea: I got up to twenty-two and then I said, “How am I gonna do this?” And then I said, “Okay, it’s still gonna be two more, so ….”
Teacher: How did you get up to twenty-two?
Andrea: I was like, okay, I added up five ….
Teacher: The five to the seventeen?
Andrea: Once I got up to twenty, I started counting two.

In the preceding task (13 and 17 beans), Andrea had first counted 3 more than 17 and then added 10. In this task, she started in the same way by counting 5 beyond 17 to get 22. It was at this point that she related the current task (15 and 17) to the previous one by keeping track of her counting acts beyond 20: “I started counting two.” That is, she added two more to the result of the previous task, 30, to get 32. Andrea’s
ability to use part-whole relations and to develop strategies strongly suggest that by the end of the teaching sessions she was constructing the explicitly nested number sequence.

In summary, many of the students’ individual solutions during the final interviews surpassed the individual solutions possible during the teaching sessions. The cooperative solutions during the teaching sessions contributed to the students’ individual cognitive progress by allowing an expansion of the range and sophistication of the students’ mathematics.

SUMMARY AND DISCUSSION

Kindergarten students working in pairs generated four strategies to organize their counting in joint counting tasks: counting parts side by side, counting all at the same time, taking turns, and working cooperatively. The four strategies required different levels of mutual coordination of actions, with counting parts side by side needing the least coordination and cooperation needing the most. As the teaching sessions progressed, all students made at least some progress toward forms of organization requiring more coordination and toward increased involvement in the partner’s actions. Two pairs of students solved counting tasks in cooperation. These students had the most advanced counting schemes; by the final interview, they had constructed or were close to constructing the initial number sequence. This apparent relationship between a student’s counting stage and the ability to work cooperatively seems reasonable if one considers some of the conditions of cooperation.

Working cooperatively requires an act of planning to establish a common goal and to project an initial action, and thus it also requires reflecting on past experiences and anticipating future ones. Reflection and anticipation are two attributes that are characteristic of students constructing the initial number sequence and that are usually not available to perceptual children within their counting activities. For perceptual children, most number words refer to “the transitory [emphasis added] experience of counting collections” (Steffe, 1992, p. 87), and no object concepts are associated with the number words. That is, on the one hand, students with only perceptual counting schemes need to count in order to give meaning to most number words. For students who are constructing the initial number sequence, on the other hand, a number word can symbolize and stand in for the activity itself as well as for the result of the activity. The ability to take a counting activity as given requires reflection on past counting experiences. This ability is paramount in cooperative counting episodes in which the counting acts of both partners merge into a single activity. In addition, students working cooperatively need to make sense of the partner’s actions. They need to be able to interpret the partner’s activity and to relate this interpretation to their own experiences, and these mutual interpretations have to be compatible. Again, the interpretation of the partner’s actions in relation to the student’s own actions requires reflection, that is, the ability to stand back and look on one’s actions from a distance.

The suggested connection between the ability to work cooperatively and a student’s counting stage is in accord with Doise and Mugny’s (1981/1984) proposition of a close and reciprocal relationship between social interaction and cognitive
development. In this relationship, “interaction enables the individual to master certain abilities which allow him to participate in more complex social interactions which in turn promotes [sic] continued cognitive development” (p. 23). The students’ cooperative organization of counting contributed to their cognitive development in that it led to a heightened awareness of the need to keep track of one’s counting acts when counting items of a screened collection. Previously, keeping track of those counting acts—for example, by enacting finger patterns—had been a mostly implicit component of a counting activity. Within the cooperative episodes, however, the act of keeping track was brought into the open and thus was made explicit.

A second manifestation of the reciprocal relationship between social interaction and cognitive development can be seen in watching and joining. The more sophisticated social interaction that occurred when a student joined the partner’s counting was an opportunity for the watching student to look at his or her “own actions from the outside and from the inside …. This may facilitate reflection on the action as an object of thought” (Sinclair, 1990, p. 22). Watching and joining also provided a basis for a student’s ability to take the partner’s counting activity as given, that is, to substitute the partner’s counting for his or her own activity. One perceptual student’s continuation of his partner’s counting activity illustrates the power of watching and joining for this student’s cognitive growth.

With respect to social interaction, watching and joining, in which a student “tuned in” to the partner’s counting activity, appeared to be a stepping stone toward cooperation. Watching the partner count afforded an opportunity to relate to the partner’s activity and to interpret this activity in reference to the student’s own counting experiences. Uttering number words in synchrony with the partner brought the partner’s activity even closer to the student’s own experiences. Watching and joining provided a frame for “taken-to-be-shared” (Wood, Cobb, Yackel, & Wheatley, 1990, p. 5) experience and knowledge.

In general, the students’ mutual involvement in the partners’ counting activities during taking turns can be seen as preparation for more sophisticated social interactions, such as cooperative solutions. As the students worked together, some cooperative behaviors identified by Cohen (1986), such as helping, listening, and being responsive to the needs of the partner, evolved implicitly, within the students’ counting activities. Of special significance for the development of such cooperative behaviors were the tasks involving unscreened collections. For the students in the perceptual counting stage and those in transition to the figurative stage, these tasks were in their “comfort zone” (J. Olive, personal communication, February 1993) but were still challenging enough to be interesting. As observers, the students could relate to their partners’ counting activity and interpret it with respect to their own experiences. These interpretations seemed to be mutually compatible and led, in turn, to increased and more sophisticated participation.

In conclusion, the interactive working condition with the students paired homogeneously rather than heterogeneously provided learning opportunities different from those in a whole-class setting, in one-to-one teaching situations, or in small-group settings with heterogeneous grouping. In the context of the teaching sessions,
working in pairs supported and enhanced the students’ cognitive development and promoted more sophisticated ways of social interaction. Students in the perceptual or figurative counting stage devised forms of participation in which they learned to relate the partner’s activity to their own counting experiences. For the students who were able to reflect on and anticipate their actions, working in pairs led to cooperative ventures in which they solved counting tasks they were unable to solve alone. Thus, when students worked in pairs, their zones of potential construction (Olive, 1994) were extended because the cooperative solutions served to neutralize perturbations in ways not within a student’s means when working individually. In turn, these cooperative solutions served as building blocks for more advanced individual constructions and solutions.

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Organization of Counting and Emergence of Cooperation

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